

Chapter 4

Linear Combinations of Vectors in \mathbb{R}^2

Definition: In \mathbb{R}^2 , \mathbf{w} is a *linear combination* of \mathbf{u} and \mathbf{v} if there exists a and b such that $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$. I let $\mathbf{u}=(u_1, u_2)$, $\mathbf{v}=(v_1, v_2)$, and $\mathbf{w}=(w_1, w_2)$. \mathbf{w} is a linear combination of \mathbf{u} and \mathbf{v} if I can solve the following two equations for a and b : $au_1 + bv_1 = w_1$ and $au_2 + bv_2 = w_2$.

In matrix form this system is

$$\begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}, \text{ or in augmented matrix form } \left(\begin{array}{cc|c} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \end{array} \right)$$

The rank of a matrix is the number of nonzero rows in a row echelon form of the matrix. Recall that this system has a unique solution iff the rank of the matrix of coefficients = 2. It has no solution if the rank of the matrix of coefficients is < 2 and rank of the augmented matrix = 2. It has many solutions if the rank of the matrix of coefficients and the augmented matrix are equal and less than two.

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Find the violations of good mathematical writing style in the above text and fix. If appropriate also use some enumeration of equations or formulas for ease of reference.